Math Weekly Lesson Preparation Guide

Teacher Name: Kimberly West	**Grade:** 11^{th} /12^{th} Precalculus
Week of: March 3^{rd} thru 7^{th}	**Unit:** 5 **Lesson Number** 5.1Fundamental Identities 5.2 Proving Trigonometric Identities

Purpose: The Weekly Lesson Preparation Guide is to provide a structure that encourages teachers to think through and internalize the daily/weekly instructional expectations.

Planning Questions	**Lesson 5.1**	**Lesson 5.2**	**Lesson 5.2**	**Remediation Make-up Day Friday**
1. Which specific Tennessee **standard(s)** are being addressed in this lesson? What is the focus of this lesson? What will the lesson **objective** be for each day?	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. Identities include : Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle. **Objective**: I can simplify trigonometric expressions using basic trigonometric identities.	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. Identities include : Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle. **Objective**: I can prove an algebraic or trigonometric identity.	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. Identities include : Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle. **Objective**: I can prove an algebraic or trigonometric identity.	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. Identities include : Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle. **P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. Identities include : Pythagorean, reciprocal, quotient, sum/difference,

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<td>Modeling:
2. Complete all tasks included in the lesson and review the sample/anticipated student responses.
For each task consider:
• What are the multiple solution paths students might take to solve this problem?
• What is the purpose of this task? Specifically, which aspect(s) of rigor are being addressed (conceptual understanding, procedural fluency, and/or application)? How does this differ based on the solution path
• Given this purpose, what key concepts and vocabulary might students need to understand to access the task?</td>
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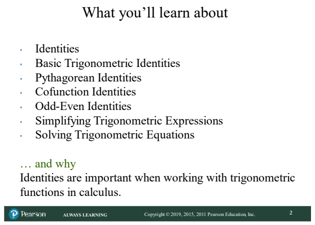

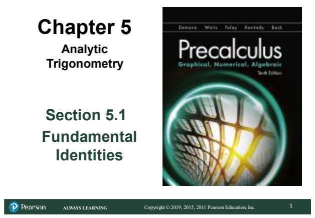

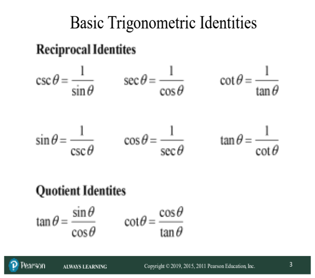

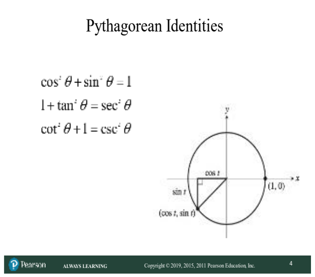

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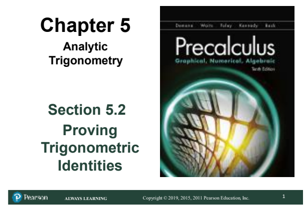

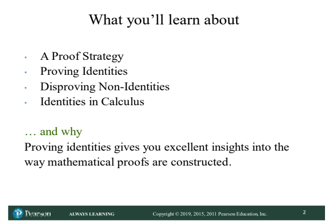

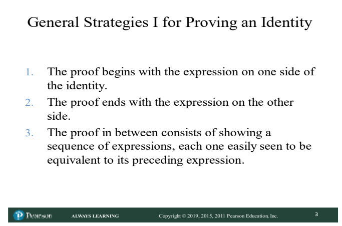

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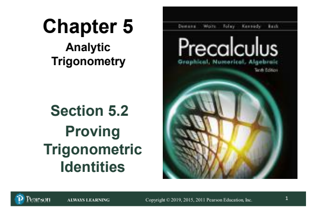

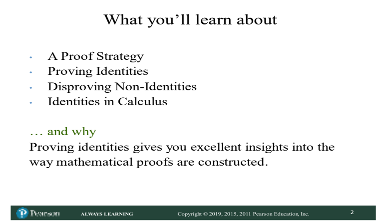

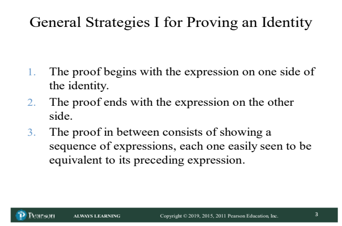

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Example 1: Using Identities

Find $\sin\theta$ and $\cos\theta$ if $\tan\theta = 3$ and $\cos\theta < 0$.

$1 + \tan^2\theta = \sec^2\theta$

$1 + 9 = \sec^2\theta$

$\sec\theta = \pm\sqrt{10}$

$\cos\theta = -1/\sqrt{10}$

To find $\sin\theta$, use $\tan\theta = 3$ and $\cos\theta = -1/\sqrt{10}$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

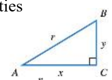
$\sin\theta = \cos\theta\tan\theta$

$\sin\theta = \left(-1/\sqrt{10}\right)(3)$

$\sin\theta = -3/\sqrt{10}$

Therefore, $\cos\theta = -1/\sqrt{10}$ and $\sin\theta = -3/\sqrt{10}$

Cofunction Identities



Angle A: $\sin A = \frac{y}{r}$ $\tan A = \frac{y}{x}$ $\sec A = \frac{r}{x}$

$\cos A = \frac{x}{r}$ $\cot A = \frac{x}{y}$ $\csc A = \frac{r}{y}$

Angle B: $\sin B = \frac{x}{r}$ $\tan B = \frac{x}{y}$ $\sec B = \frac{r}{y}$

$\cos B = \frac{y}{r}$ $\cot B = \frac{y}{x}$ $\csc B = \frac{r}{x}$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$

Even-Odd Identities

$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$

$\csc(-x) = -\csc x$ $\sec(-x) = \sec x$ $\cot(-x) = -\cot x$

General Strategies II for Proving an Identity

1. Begin with the more complicated expression and work toward the less complicated expression.
2. If no other move suggests itself, convert the entire expression to one involving sines and cosines.
3. Combine fractions by combining them over a common denominator.

General Strategies III for Proving an Identity

1. Use the algebraic identity $(a + b)(a - b) = a^2 - b^2$ to set up applications of the Pythagorean identities.
2. Always be mindful of the "target" expression, and favor manipulations that bring you closer to your goal.

Identities in Calculus

1. $\cos^3 x = \left(1 - \sin^2 x\right)(\cos x)$
2. $\sec^4 x = \left(1 + \tan^2 x\right)\left(\sec^2 x\right)$
3. $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$
4. $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$
5. $\sin^5 x = \left(1 - 2\cos^2 x + \cos^4 x\right)(\sin x)$
6. $\sin^2 x\cos^5 x = \left(\sin^2 - 2\sin^4 x + \sin^6 x\right)(\cos x)$

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Example: Simplifying by Factoring and Using Identities

Simplify the expression $\cos^3 x + \cos x \sin^2 x$.

$$\begin{aligned}\cos^3 x + \cos x \sin^2 x &= \cos x(\cos^2 x + \sin^2 x)\\ &= \cos x(1) \quad \text{Pythagorean Identity}\\ &= \cos x\end{aligned}$$

Example: Simplifying by Expanding and Using Identities

Simplify the expression: $\dfrac{(\csc x - 1)(\csc x + 1)}{\cos^2 x}$

$$\begin{aligned}\frac{(\csc x - 1)(\csc x + 1)}{\cos^2 x} &= \frac{\csc^2 x - 1}{\cos^2 x} \quad (a+b)(a-b) = a^2 - b^2\\ &= \frac{\cot^2 x}{\cos^2 x} \quad \text{Pythagorean Identity}\\ &= \frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}\\ &= \frac{1}{\sin^2 x}\\ &= \csc^2 x\end{aligned}$$

Example : Solving a Trigonometric Equation (1 of 2)

Find all values of x in the interval $[0, 2\pi)$ that solve

$$\frac{\sin^3 x}{\cos x} = \tan x.$$

Example:

Find $\sin\theta$ and $\cos\theta$ if $\tan\theta = 3$ and $\cos\theta < 0$.

SOLUTION

We could solve this problem by the reference triangle techniques of Section 4.3 (see Alternate Example 7 of that section), but we will show an alternate solution here using identities.

First we note that $\sec^2\theta = 1 + \tan^2\theta = 1 + 3^2 = 10$, so $\sec\theta = \pm\sqrt{10}$.

Since $\sec\theta = \pm\sqrt{10}$, we have $\cos\theta = 1/\sec\theta = 1/\pm\sqrt{10}$.

But $\cos\theta < 0$, so $\cos\theta = -\sqrt{10}$.

Finally,

$$\begin{aligned}\tan\theta &= 3\\ \frac{\sin\theta}{\cos\theta} &= 3\\ \sin\theta = 3\cos\theta &= 3\left(-\frac{1}{\sqrt{10}}\right) = -\frac{3}{\sqrt{10}}\end{aligned}$$

Therefore, $\sin\theta = -\dfrac{3}{\sqrt{10}}$ and $\cos\theta = -\dfrac{1}{\sqrt{10}}$.

Example: Solving a Trigonometric Equation

Find all values of x in the interval $[0, 2\pi)$ that solve $\sin^3 x/\cos x = \tan x$.

SOLUTION

$$\begin{aligned}\frac{\sin^3 x}{\cos x} &= \tan x\\ \frac{\sin^3 x}{\cos x} &= \frac{\sin x}{\cos x}\\ \sin^3 x &= \sin x \quad \text{Multiply both sides by } \cos x.\\ \sin^3 x - \sin x &= 0\\ (\sin x)(\sin^2 x - 1) &= 0\\ (\sin x)(-\cos^2 x) &= 0 \quad \text{Pythagorean identity}\\ \sin x = 0 &\text{ or } \cos x = 0\end{aligned}$$

We reject the possibility that $\cos x = 0$ because that would make both sides of the original equation undefined.

The values in the interval $[0, 2\pi)$ that solve $\sin x = 0$ (and therefore $\sin^3 x/\cos x = \tan x$) are 0 and π.

Example: Proving an Identity Useful in Calculus

Prove the following identity:

$$\begin{aligned}\sin^5 x\cos^2 x &= (\sin x)(\cos^2 - 2\cos^4 x + \cos^6 x)\\ &= (\sin x)(\sin^2 x)^2(\cos^2 x)\\ &= (\sin x)(1 - \cos^2 x)^2(\cos^2 x)\\ &= (\sin x)(1 - 2\cos^2 x + \cos^4 x)(\cos^2 x)\\ &= (\sin x)(\cos^2 - 2\cos^4 x + \cos^6 x)\end{aligned}$$

Example: Proving an Identity

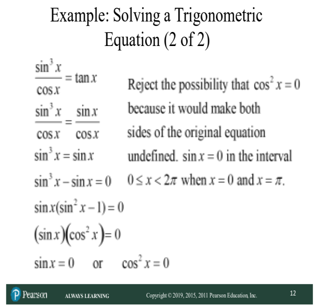
Prove the identity: $\sin x - \csc x = -\cos x \cot x$.

SOLUTION

We begin by deciding whether to start with the expression on the right or the left. It is usually best to start with the more complicated expression, as it is easier to proceed from the complex toward the simple than to go in the other direction. The expression on the left is slightly more complicated because it involves two terms.

$$\begin{aligned}\sin x - \csc x &= \sin x - \frac{1}{\sin x}\\ &= \frac{\sin^2 x}{\sin x} - \frac{1}{\sin x} \quad \text{Setting up a common denominator}\\ &= \frac{\sin^2 x - 1}{\sin x}\\ &= \frac{-\cos^2 x}{\sin x} \quad \text{Pythagorean identity}\\ &= -\cos x\left(\frac{\cos x}{\sin x}\right)\\ &= -\cos x \cot x \quad \text{Basic identities}\end{aligned}$$

(Remember that the "floaters" are not part of the proof.)

	 **Vocabulary** • Identities • Domain of Validity	**Vocabulary** • Reduction Formula	**Vocabulary** • Reduction Formula	
3. What **specific tasks/problems** will you use to reveal understanding of the grade-level standard(s)? (refer to the Instructional Focus Document Evidence of Learning Statements)	*Selective Practice Problems from pages **403-404** *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	*Selective Practice Problems from pages **411-412** *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	*Selective Practice Problems from pages **411-412** *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	
Additional Considerations				
If your lesson contains homework, how will you utilize the work? Will you need to send scaffolding notes home? Is there a strategy you can use to maximize homework?		**Homework will be utilized by:** **Align with Learning Objectives:** Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. **Variety of Tasks:** Include different types of problems (e.g., practice, application, extension) to cater to various levels of understanding and to reinforce the concept from multiple angles. **Scaffolded Problems:** Start with easier problems and gradually increase difficulty. This helps	**Homework will be utilized by:** **Align with Learning Objectives:** Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. **Variety of Tasks:** Include different types of problems (e.g., practice, application, extension) to cater to various levels of understanding and to reinforce the concept from multiple angles. **Scaffolded Problems:** Start with easier problems and gradually increase difficulty. This helps build	

		build confidence and understanding before tackling more complex tasks. Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.	confidence and understanding before tackling more complex tasks. Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.	