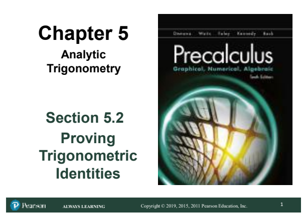
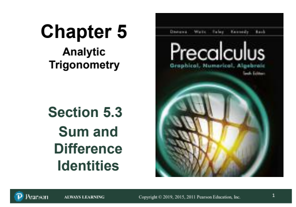
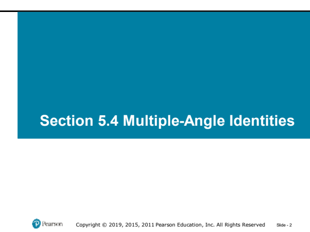
Math Weekly Lesson Preparation Guide

Teacher Name: Kimberly West	**Grade:** 11th /12th Precalculus
Week of: March 17th thru 21st	**Unit:** 5 **Lesson Number:** 2 , 3 and 4

Purpose: The Weekly Lesson Preparation Guide is to provide a structure that encourages teachers to think through and internalize the daily/weekly instructional expectations.

Planning Questions	**Lesson 5.2**	**Lesson 5.3**	**Lesson 5.4**	**Remediation Make-up Day Friday**
1. Which specific Tennessee **standard(s)** are being addressed in this lesson? What is the focus of this lesson? What will the lesson **objective** be for each day?	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. **Objective**: I can use trigonometric identities to solve problems.	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. **Objective**: I can prove an algebraic or trigonometric identity.	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. **Objective**: I can use trigonometric identities to solve problems.	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. Identities include : Pythagorean, reciprocal, quotient, sum/difference,
Modeling: 2. **Complete all tasks** included in the lesson and review the sample/anticipated student responses.				

For each task consider:

- What are the multiple solution paths students might take to solve this problem?
- What is the purpose of this task? Specifically, which aspect(s) of rigor are being addressed (conceptual understanding, procedural fluency, and/or application)? How does this differ based on the solution path
- Given this purpose, what key concepts and vocabulary might students need to understand to access the task?

What you'll learn about

- A Proof Strategy
- Proving Identities
- Disproving Non-Identities
- Identities in Calculus

… and why

Proving identities gives you excellent insights into the way mathematical proofs are constructed.

General Strategies I for Proving an Identity

1. The proof begins with the expression on one side of the identity.
2. The proof ends with the expression on the other side.
3. The proof in between consists of showing a sequence of expressions, each one easily seen to be equivalent to its preceding expression.

General Strategies II for Proving an Identity

1. Begin with the more complicated expression and work toward the less complicated expression.
2. If no other move suggests itself, convert the entire expression to one involving sines and cosines.
3. Combine fractions by combining them over a common denominator.

General Strategies III for Proving an Identity

1. Use the algebraic identity $(a+b)(a-b) = a^2 - b^2$ to set up applications of the Pythagorean identities.
2. Always be mindful of the "target" expression, and favor manipulations that bring you closer to your goal.

What you'll learn about

- Cosine of a Difference
- Cosine of a Sum
- Sine of a Difference or Sum
- Tangent of a Difference or Sum
- Verifying a Sinusoid Algebraically

… and why

These identities provide clear examples of how different the algebra of functions can be from the algebra of real numbers.

Cosine of a Sum or Difference

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

(Note the sign switch in either case.)

Example . Using the Cosine-of-a-Difference Identity

Find the exact value of $\cos\ 75^\circ$ without using a calculator.

$$\begin{aligned}\cos\ 75^\circ &= \cos\ (45^\circ + 30^\circ)\\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ\\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)\\ &= \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

Sine of a Sum or Difference

$$\sin(u \pm v) = \sin u \cos v \pm \sin v \cos u$$

(Note that the sign does *not* switch in either case.)

What You'll Learn About

- Double-Angle Identities
- Power-Reducing Identities
- Half-Angle Identities
- Solving Trigonometric Equations

… and why

These identities are useful in calculus courses.

Double Angle Identities

$$\sin 2u = 2\sin u \cos u$$

$$\cos 2u = \begin{cases}\cos^2 u - \sin^2 u\\ 2\cos^2 u - 1\\ 1 - 2\sin^2 u\end{cases}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

Example: Proving a Double-Angle Identity

$$\begin{aligned}\cos 2x &= \cos(x+x)\\ &= \cos x \cos x - \sin x \sin x\\ &= \cos^2 x - \sin^2 x\end{aligned}$$

Power-Reducing Identities

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Identities in Calculus

1. $\cos^3 x = (1-\sin^2 x)(\cos x)$
2. $\sec^4 x = (1+\tan^2 x)(\sec^2 x)$
3. $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$
4. $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$
5. $\sin^5 x = (1-2\cos^2 x + \cos^4 x)(\sin x)$
6. $\sin^2 x \cos^5 x = (\sin^2 - 2\sin^4 x + \sin^6 x)(\cos x)$

Example:

Find $\sin\theta$ and $\cos\theta$ if $\tan\theta = 3$ and $\cos\theta < 0$.

SOLUTION

We could solve this problem by the reference triangle techniques of Section 4.3 (see Alternate Example 7 of that section), but we will show an alternate solution here using identities.

First we note that $\sec^2\theta = 1 + \tan^2\theta = 1 + 3^2 = 10$, so $\sec\theta = \pm\sqrt{10}$.

Since $\sec\theta = \pm\sqrt{10}$, we have $\cos\theta = 1/\sec\theta = 1/\pm\sqrt{10}$.

But $\cos\theta < 0$, so $\cos\theta = -\sqrt{10}$.

Finally,

$$\tan\theta = 3$$
$$\frac{\sin\theta}{\cos\theta} = 3$$
$$\sin\theta = 3\cos\theta = 3\left(-\frac{1}{\sqrt{10}}\right) = -\frac{3}{\sqrt{10}}$$

Therefore, $\sin\theta = -\frac{3}{\sqrt{10}}$ and $\cos\theta = -\frac{1}{\sqrt{10}}$.

Example: Solving a Trigonometric Equation

Find all values of x in the interval $[0, 2\pi)$ that solve $\sin^3 x/\cos x = \tan x$.

SOLUTION

$$\frac{\sin^3 x}{\cos x} = \tan x$$
$$\frac{\sin^3 x}{\cos x} = \frac{\sin x}{\cos x}$$
$$\sin^3 x = \sin x \quad \text{Multiply both sides by } \cos x.$$
$$\sin^3 x - \sin x = 0$$
$$(\sin x)(\sin^2 x - 1) = 0$$
$$(\sin x)(-\cos^2 x) = 0 \quad \text{Pythagorean identity}$$
$$\sin x = 0 \text{ or } \cos x = 0$$

We reject the possibility that $\cos x = 0$ because that would make both sides of the original equation undefined.

The values in the interval $[0, 2\pi)$ that solve $\sin x = 0$ (and therefore $\sin^3 x/\cos x = \tan x$) are 0 and π.

Vocabulary

- Identities
- Domain of Validity

Example . Using the Sum and Difference Formulas

Write the following expression as the sine or cosine of an angle: $\sin\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{3}$

Recognize $\sin\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{3}$ as the $\sin(u+v)$.

$$\sin\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{3} = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin\frac{7\pi}{12}$$

Tangent of a Difference of Sum

$$\tan(u \pm v) = \frac{\sin(u \pm v)}{\cos(u \pm v)} = \frac{\sin u\cos v \pm \sin v\cos u}{\cos u\cos v \mp \sin u\sin v}$$

or

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u\tan v}$$

Example: Expressing a Sum of Sinusoids as a Sinusoid

Find values for a, b, and h so that for all x,
$$4\cos 2x - 7\sin 2x = a\sin\left[b(x-h)\right].$$

Example: Expressing a Sum of Sinusoids as a Sinusoid

$$4\cos 2x - 7\sin 2x = a\sin\left[b(x-h)\right]$$

is a sinusoid with period $\frac{2\pi}{2} = \pi$, so $b = 2$

$$4\cos 2x - 7\sin 2x = a\sin\left[2(x-h)\right]$$
$$= a\sin(2x - 2h)$$
$$= a\sin 2x\cos 2h - a\cos 2x\sin 2h$$
$$4\cos 2x - 7\sin 2x = (-a\sin 2h)\cos 2x + (a\cos 2h)\sin 2x,$$

so $4 = -a\sin 2h$ and $-7 = a\cos 2h$

Example: Reducing a Power of 4 (1 of 2)

Rewrite $\sin^4 x$ in terms of trigonometric functions with no power greater than 1.

Example: Reducing a Power of 4 (2 of 2)

$$\sin^4 x = (\sin^2 x)^2$$
$$= \left(\frac{1-\cos 2x}{2}\right)^2$$
$$= \frac{1 - 2\cos 2x + \cos^2 2x}{4}$$
$$= \frac{1}{4} - \frac{\cos 2x}{2} + \frac{1}{4}\left(\frac{1+\cos 4x}{2}\right)$$
$$= \frac{1}{4} - \frac{\cos 2x}{2} + \frac{1+\cos 4x}{8}$$

Half-Angle Identities

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

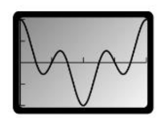
$$\tan\frac{u}{2} = \begin{cases} \pm\sqrt{\dfrac{1-\cos u}{1+\cos u}} \\ \dfrac{1-\cos u}{\sin u} \\ \dfrac{\sin u}{1+\cos u} \end{cases}$$

Example: Using a Double Angle Identity (1 of 3)

Solve $\cos x + \cos 3x = 0$ in the interval $[0, 2\pi)$.

Solve Graphically

The graph suggest that there are six solutions: 0.79, 1.57, 2.36, 3.93, 4.71, 5.50.



$[0, 2\pi]$ by $[-2, 2]$

Example: Expressing a Sum of Sinusoids as a Sinusoid

so $4 = -a\sin 2h$ and $-7 = a\cos 2h$

$\sin 2h = -\frac{4}{a}$ and $\cos 2h = -\frac{7}{a}$

so $\tan 2h = \frac{-4/a}{-7/a} = \frac{4}{7}$

so possible values of $2h$ are $\tan^{-1}\left(\frac{4}{7}\right) + n\pi$

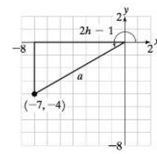
Example: Expressing a Sum of Sinusoids as a Sinusoid

To find a, we use $\sin 2h = -\frac{4}{a}$

and $\cos 2h = -\frac{7}{a}$

Because a is the hypotenuse,

$$a = \sqrt{(-7)^2 + (-4)^2} = \sqrt{65}$$



Thus,

$$4\cos 2x - 7\sin 2x = \sqrt{65}\sin\left\{2\left[x - \frac{1}{2}\tan^{-1}\left(\frac{4}{7}\right) - \frac{\pi}{2}\right]\right\}$$

Example: Expressing a Sum of Sinusoids as a Sinusoid

Support Graphically

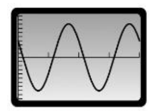
The graphs of

$y_1 = 4\cos 2x - 7\sin 2x$

and

$y_2 = \sqrt{65}\sin\left\{2\left[x - \frac{1}{2}\tan^{-1}\left(\frac{4}{7}\right) - \frac{\pi}{2}\right]\right\}$

are identical.



$[0, 2\pi]$ by $[-10, 10]$

Example: Expressing a Sum of Sinusoids as a Sinusoid

Interpret

The values are $a = \sqrt{65}$, $b = 2$, and $h = \frac{1}{2}\tan^{-1}\left(\frac{4}{7}\right) + \frac{\pi}{2}$

(other values are possible).

The amplitude is $\sqrt{65}$, the peridod is π, and

the phase shift is $\frac{1}{2}\tan^{-1}\left(\frac{4}{7}\right) + \frac{\pi}{2}$ radians.

Example: Using a Double Angle Identity (2 of 3)

Solve $\cos x + \cos 3x = 0$ in the interval $[0, 2\pi)$.

Confirm Algebraically

$$\cos x + \cos 3x = 0$$
$$\cos x + \cos x\cos 2x - \sin x\sin 2x = 0$$
$$\cos x + \cos x\left(1 - 2\sin^2 x\right) - \sin x\left(2\sin x\cos x\right) = 0$$
$$\cos x + \cos x - 2\cos x\sin^2 x - 2\cos x\sin^2 x = 0$$
$$2\cos x - 4\cos x\sin^2 x = 0$$
$$2\cos x\left(1 - 2\sin^2 x\right) = 0$$

Example: Using a Double Angle Identity (3 of 3)

$2\cos x(1 - 2\sin^2 x) = 0$

$\cos x = 0$ or $1 - 2\sin^2 x = 0$

$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ or $\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ or $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, or $\frac{7\pi}{4}$

The six exact solutions in the given interval are

$\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}$, and $\frac{7\pi}{4}$.

3. What **specific tasks/problems** will you use to reveal understanding of the grade-level standard(s)? (refer to the Instructional Focus Document Evidence of Learning Statements)	*Selective Practice Problems from pages **411-412** *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	*Selective Practice Problems from pages **418 - 419** *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	*Selective Practice Problems from page **425** *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	
Additional Considerations				
If your lesson contains homework, how will you utilize the work? Will you need to send scaffolding notes home? Is there a strategy you can use to maximize homework?	**Homework will be utilized by:** **Align with Learning Objectives:** Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. **Variety of Tasks:** Include different types of problems (e.g., practice, application, extension) to cater to various levels of understanding and to reinforce the concept from multiple angles. **Scaffolded Problems:** Start with easier problems and gradually increase difficulty. This helps build confidence and understanding before tackling more complex tasks. Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.	**Homework will be utilized by:** **Align with Learning Objectives:** Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. **Variety of Tasks:** Include different types of problems (e.g., practice, application, extension) to cater to various levels of understanding and to reinforce the concept from multiple angles. **Scaffolded Problems:** Start with easier problems and gradually increase difficulty. This helps build confidence and understanding before tackling more complex tasks. Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.	**Homework will be utilized by:** **Align with Learning Objectives:** Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. **Variety of Tasks:** Include different types of problems (e.g., practice, application, extension) to cater to various levels of understanding and to reinforce the concept from multiple angles. **Scaffolded Problems:** Start with easier problems and gradually increase difficulty. This helps build confidence and understanding before tackling more complex tasks. Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.	