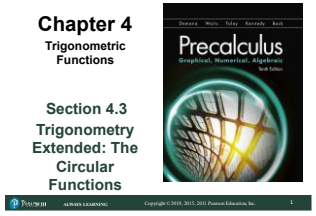
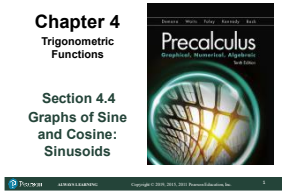
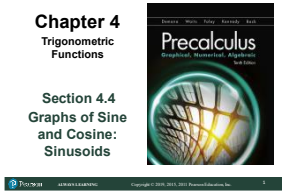
Math Weekly Lesson Preparation Guide

Teacher Name: Kimberly West	**Grade:** 11th /12th Precalculus
Week of: January 27th thru 31st	**Unit:** 4 **Lesson Numbers:** 4.3 Periodic Functions and 4.4 Graphs of Sine and Cosine

Purpose: The Weekly Lesson Preparation Guide is to provide a structure that encourages teachers to think through and internalize the daily/weekly instructional expectations.

Planning Questions	**Monday Assessment QUIZ**	**Tuesday Lesson 4.3**	**Wednesday Lesson 4.4**	**Thursday Lesson 4.4**	**Assessment OR Remediation**
1. Which specific Tennessee **standard(s)** are being addressed in this lesson? What is the focus of this lesson? What will the lesson **objective** be for each day?	**P.F.TF.A.2** Convert from radians to degrees and from degrees to radians **P.F.TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **P.G.AT.A.1** Use the definitions of the six trigonometric ratios as ratios of the sides in a right tringle to solve problems about lengths of sides and measures of angles. **P.F.TF.A.3** Use special triangles to determine geometrically the values	**P.F.TF.A.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions **Objective**: I can use periodicity to evaluate trig functions	**P.F.GT.A.3** Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes **P.F.GT.A.1** Interpret transformations of trigonometric functions **Objective**: I can graph sinusoids.	**P.F.GT.A.3** Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes **Objective**: I can find the amplitude, period, and/or frequency of a transformed sine or cosine function	**P.F.TF.A.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions **P.F.GT.A.1** Interpret transformations of trigonometric functions **P.F.GT.A.3** Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase

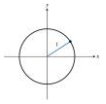
	of sine, cosine, tangent for π/3, π/4, and π/6, and explain how to use the unit circle to express the values of sine, cosine, and tangent for π-x, π+x, and 2π-x in terms of their values for x, where x is any real number **P.G.AT.A.1** Use the definitions of the six trigonometric ratios as ratios of the sides in a right tringle to solve problems about lengths of sides and measures of angles. **P.F.TF.A.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions				shift, and asymptotes
Modeling: 2. **Complete all tasks** included in the lesson and review the sample/anticipated student responses. For each task consider: • What are the multiple solution paths students might					

take to solve this problem?

- What is the purpose of this task? Specifically, which aspect(s) of rigor are being addressed (conceptual understanding, procedural fluency, and/or application)? How does this differ based on the solution path
- Given this purpose, what key concepts and vocabulary might students need to understand to access the task?

Unit Circle

The unit circle is a circle of radius 1 centered at the origin.



Trigonometric Functions of Real Numbers

Let t be any real number, and let $P(x, y)$ be the point corresponding to t when the number line is wrapped onto the unit circle as described above. Then

$\sin t = y$ $\quad \csc t = \frac{1}{y} \quad (y \neq 0)$

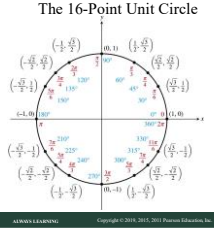
$\cos t = x$ $\quad \sec t = \frac{1}{x} \quad (x \neq 0)$

$\tan t = \frac{y}{x} \quad (x \neq 0)$ $\quad \cot t = \frac{x}{y} \quad (y \neq 0)$

Periodic Function

A function $y = f(t)$ is **periodic** if there is a positive number c such that $f(t + c) = f(t)$ for all values of t in the domain of f. The smallest such number c is called the **period** of the function.

The 16-Point Unit Circle



Sinusoid

A function is a **sinusoid** if it can be written in the form $f(x) = a\sin(bx + c) + d$ where a, b, c, and d are constants and neither a nor b is 0.

Amplitude of a Sinusoid

The **amplitude** of the sinusoid $f(x) = a\sin(bx + c) + d$ is $|a|$.
Similarly, the amplitude of $f(x) = a\cos(bx + c) + d$ is $|a|$.
Graphically, the amplitude is half the height of the wave.

Period of a Sinusoid

The **period** of the sinusoid $f(x) = a\sin(bx + c) + d$ is $2\pi/|b|$.
Similarly, the period of $f(x) = a\cos(bx + c) + d$ is $2\pi/|b|$.
Graphically, the period is the length of one full cycle of the wave.

Example: Horizontal Stretch or Shrink and Period

Find the period of $y = \sin\left(\frac{x}{2}\right)$ and use the language of transformations to describe how the graph relates to $y = \sin x$.

The period is $\frac{2\pi}{1} = 4\pi$. The graph of $y = \sin\left(\frac{x}{2}\right)$ is a horizontal stretch of $y = \sin x$ by a factor of 2.

Frequency of a Sinusoid

The **frequency** of the sinusoid $f(x) = a\sin(bx + c) + d$ is $|b|/2\pi$.
Similarly, the frequency of $f(x) = a\cos(bx + c) + d$ is $|b|/2\pi$.
Graphically, the frequency is the number of complete cycles the wave completes in a unit interval.

Example: Combining a Phase Shift with a Period Change

Construct a sinusoid with period $\pi/3$ and amplitude 4 that goes through $(2, 0)$.
To find the coefficient of x, set $2\pi/|b| = \pi/3$ and solve for b.
Find $b = \pm 6$. Arbitrarily choose $b = 6$.
For the amplitude set $|a| = 4$. Arbitrarily choose $a = 4$.
The graph contains (2,0) so shift the function 2 units to the right.

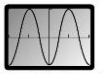
$y = 4\sin(6(x - 2)) = 4\sin(6x - 12)$.

Example: Combining a Phase Shift with a Period Change

Find the frequency of the function $f(x) = -\frac{1}{3}\cos 5x$ and interpret its meaning graphically. Sketch the graph in the window $\left[-\frac{2\pi}{5}, \frac{2\pi}{5}\right]$ by $\left[-\frac{1}{3}, \frac{1}{3}\right]$.

Solution

The frequency is $5 \div 2\pi = \frac{5}{2\pi}$.
This is the reciprocal of the period, which is $2\pi/5$.
The graph completes one cycle per interval of length $2\pi/5$.



$\left[-\frac{2\pi}{5}, \frac{2\pi}{5}\right]$ by $\left[-\frac{1}{3}, \frac{1}{3}\right]$

Graphs of Sinusoids (1 of 2)

The graphs of $y = a\sin(b(x - h)) + k$ and $y = a\cos(b(x - h)) + k$ (where $a \neq 0$ and $b \neq 0$) have the following characteristics

$\text{amplitude} = |a|$

$\text{period} = \frac{2\pi}{|b|}$

$\text{frequency} = \frac{|b|}{2\pi}$

		Vocabulary • Initial Side • Vertex • Terminal Side • Measure of an Angle • Positive Angle • Negative Angle • Standard Position • Coterminal Angles • Reference Triangle • Reference Angle • Quadrantal Angles • Unit Circle • Wrapping Function • Circular Functions • Periodic • Period	**Vocabulary** • Sinusoid • Amplitude • Frequency • Phase Shift	Graphs of Sinusoids (2 of 2) When compared to the graphs of $y = a\sin bx$ and $y = a\cos bx$, respectively, they also have the following characteristics: a phase shift of h; a vertical translation of k.  Constructing a Sinusoidal Model Using Time (1 of 2) 1. Determine the maximum value M and minimum value m. The amplitude A of the sunusoid will be $A = \frac{M-m}{2}$, and the vertical shift will be $C = \frac{M+m}{2}$. 2. Determine the period p, the time interval of a single cycle of the periodic function. The horizontal shrink (or stretch) will be $B = \frac{2\pi}{p}$.  Constructing a Sinusoidal Model using Time (2 of 2) 3. Choose an appropriate sinusoid based on behavior at some given time T. For example, at time T: $f(t) = A\cos(B(t-T)) + C$ attains a maximum value; $f(t) = -A\cos(B(t-T)) + C$ attains a minimum value; $f(t) = A\sin(B(t-T)) + C$ is halfway between a minimum and a maximum value; $f(t) = -A\sin(B(t-T)) + C$ is halfway between a maximum and a minimum value.  **Vocabulary** • Sinusoid • Amplitude • Frequency • Phase Shift	

3. What **specific tasks/problems** will you use to reveal understanding of the grade-level standard(s)? (refer to the Instructional Focus Document Evidence of Learning Statements)		*Selective Practice Problems from pages **340-341** *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	*Selective Practice Problems from pages **350-341** *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	*Selective Practice Problems from pages **350-341** *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	
Additional Considerations					
If your lesson contains homework, how will you utilize the work? Will you need to send scaffolding notes home? Is there a strategy you can use to maximize homework?		**Homework will be utilized by:** **Align with Learning Objectives:** Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. **Variety of Tasks:** Include different types of problems (e.g., practice, application, extension) to cater to various levels of understanding and to reinforce the concept from multiple angles. **Scaffolded Problems:** Start with easier problems and gradually increase difficulty. This helps build confidence and understanding before tackling more complex tasks. Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.	**Homework will be utilized by:** **Align with Learning Objectives:** Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. **Variety of Tasks:** Include different types of problems (e.g., practice, application, extension) to cater to various levels of understanding and to reinforce the concept from multiple angles. **Scaffolded Problems:** Start with easier problems and gradually increase difficulty. This helps build confidence and understanding before tackling more complex tasks. Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.	**Homework will be utilized by:** **Align with Learning Objectives:** Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. **Variety of Tasks:** Include different types of problems (e.g., practice, application, extension) to cater to various levels of understanding and to reinforce the concept from multiple angles. **Scaffolded Problems:** Start with easier problems and gradually increase difficulty. This helps build confidence and understanding before tackling more complex tasks. Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.	